The Structure of Information Networks

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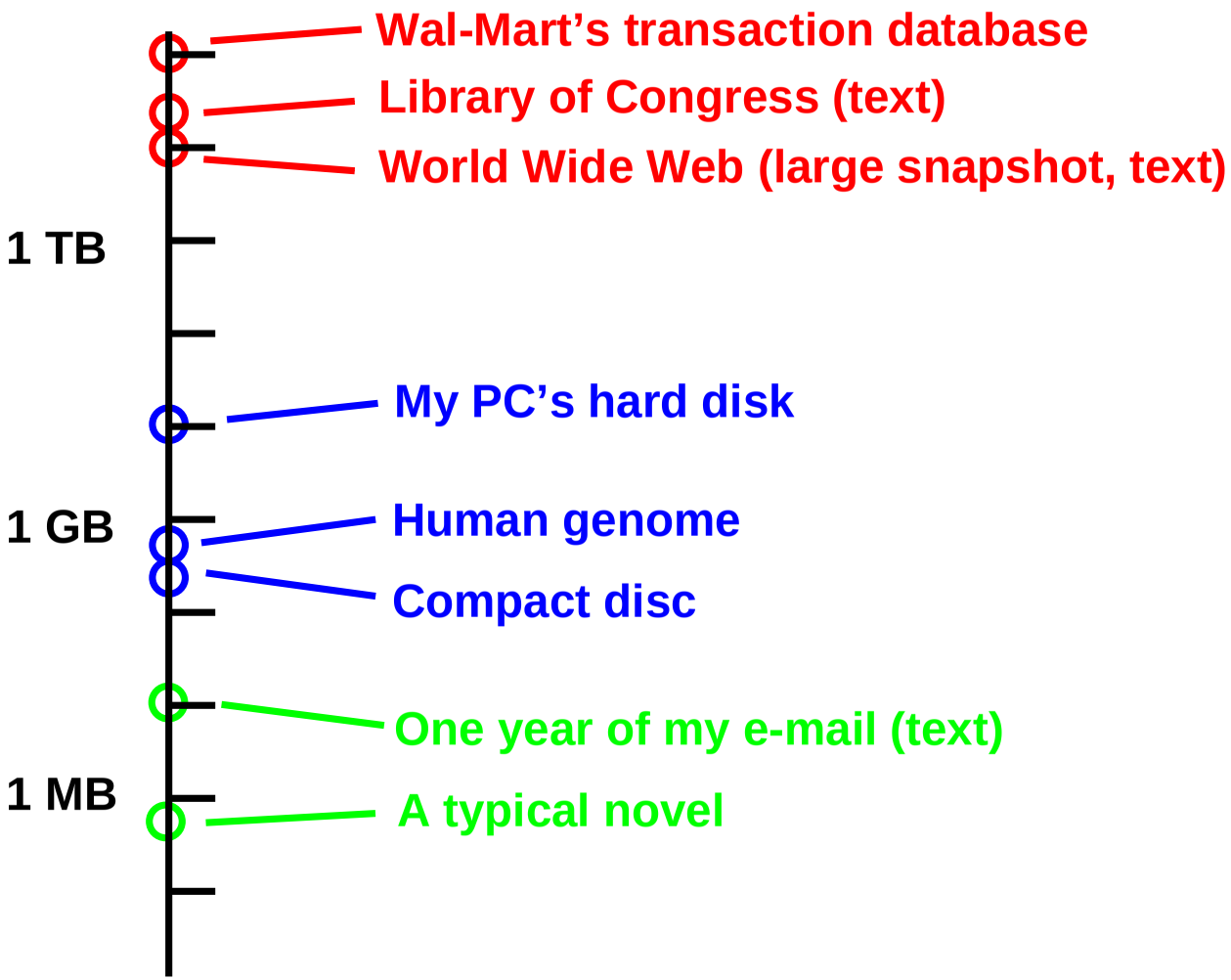
How much information is there?

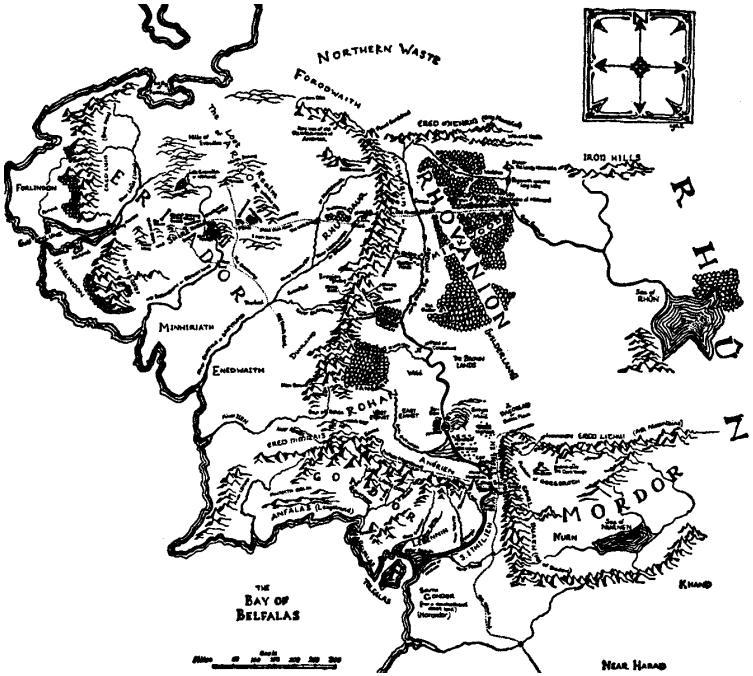


Information Networks

The information we deal with is taking on a networked character.

- **Storage and indexing of massive datasets**
  - ▷ **Scientific and patent citation databases.**
  - ▷ **Social networks and collaboration graphs.**
  - ▷ **The phone call graph.**
  - ▷ **Consumer preferences, collaborative filtering.**

- **Creation of content with an intrinsic link structure**
  - ▷ **The World Wide Web.**



The emergence of ‘cyberspace’ and the

World Wide Web is like the discovery of a new continent.

– Jim Gray, 1998 Turing Award address

Complex networks as phenomena — not just technological artifacts.

What structure emerges?

How can we exploit it to understand the underlying content?

Network Structure

- **Unit Structure**

  A single node's degree: the number of links to it.

  What is the distribution of node degrees?

- **Local Structure**

  What does the neighborhood of a node look like?

  What small structures recur in the Web's link topology?

- **Global Structure**

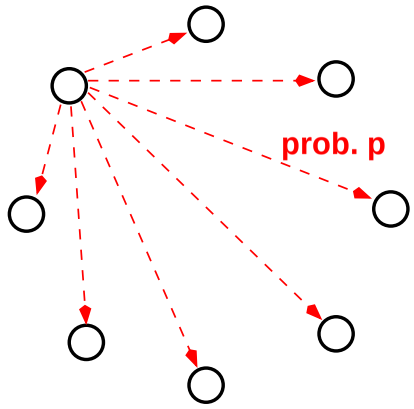
  How well connected is the link structure?

  What is the typical distance between nodes?

Generative Models

Comparing observed structure with predictions of generative models.

- **To what extent can structure be explained by simple models?**
- **Network models as a testbed for designing heuristics.**



Traditional model for randomly generated graphs [Erdös-Rényi 1960]

Start with a fixed set of nodes.

Link v to w independently with probability p.

First observation: degrees strongly concentrated around average.

Very high degrees exponentially unlikely.

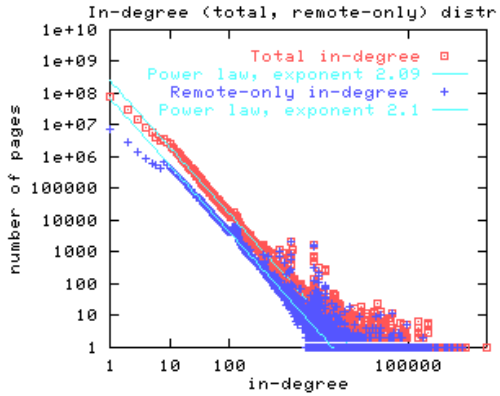
Power Laws

Fraction of Web pages with n in-links is $1/n^{2.1}$ ($\longrightarrow$ relatively likely).

[Barabasi et al. 1999], [Broder et al. 2000], [Huberman et al. 1999]

Fraction of Internet domains with n neighbors is $1/n^{2.15}$

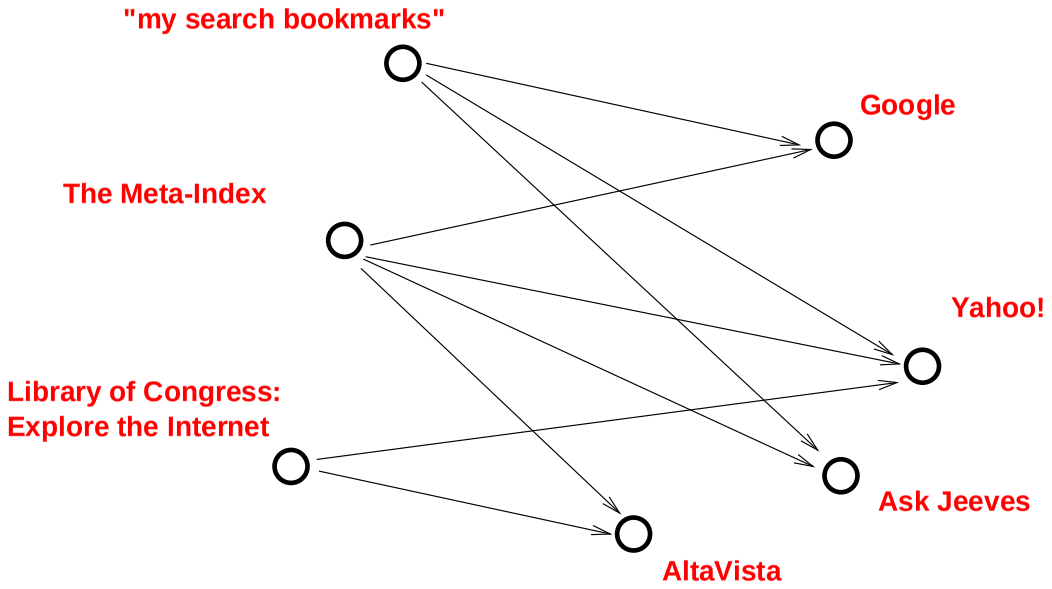
[Faloutsos et al. 1999, incl. several other Internet power laws]



Power laws appear throughout socially constructed systems:

- **Zipf [1949]: The n^{th} most frequent word occurs at rate $1/n$.**
- **Lotka [1926]: Fraction of authors with n papers is $1/n^2$.**
- **Pareto [1897]: Wealth distribution follows a power law.**

Local Structure



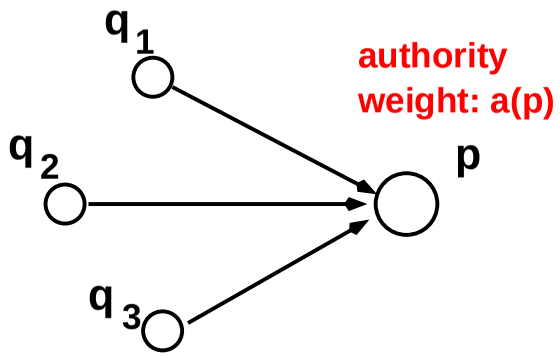
Density of link structure frequently signifies a coherent topic.

The core of a hyperlinked "community" is an inter-linked set of hubs and authorities [Kleinberg 1998].

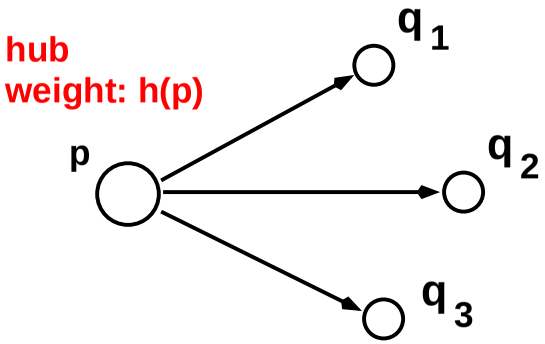
A hub is a page that links to many good authorities.

An authority is a page that is linked to by many good hubs.

Finding Hubs and Authorities



$$a(p) \leftarrow \sum_{q \rightarrow p} h(q)$$



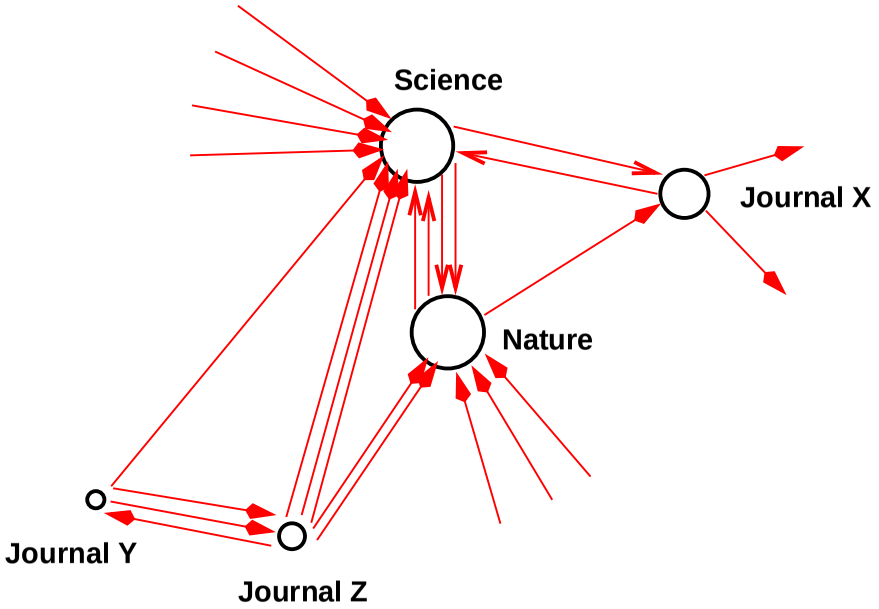
$$h(p) \leftarrow \sum_{p \rightarrow q} a(q)$$

Theorem: Repeated updating operations converge to a fixed point.

Limiting vectors of weights a^* and h^* are

eigenvectors of "symmetrized" adjacency matrices $A^T A$ and $A A^T$.

Bibliometrics, . . .



Measures of centrality in social networks [Katz 1953], [Hubbell 1965].

Garfield, 1972: The Impact Factor. Normalized citation counting.

Pinski-Narin, 1976: Influence Weights

- **Influential journals are cited, recursively, by influential journals.**
- **Propagation: "authorities" directly reinforce other "authorities."**

Random Walks, . . . and Web Search

Brin and Page, 1998.

- **How should we model Web browsing?**

  User repeatedly follows six random links, then takes random jump.

- **Equilibrium of walk is "one-level" weight-propagation scheme.**

  Simplified variation of hub-authority computation, with no hubs.

  Can be used to rank Web pages.

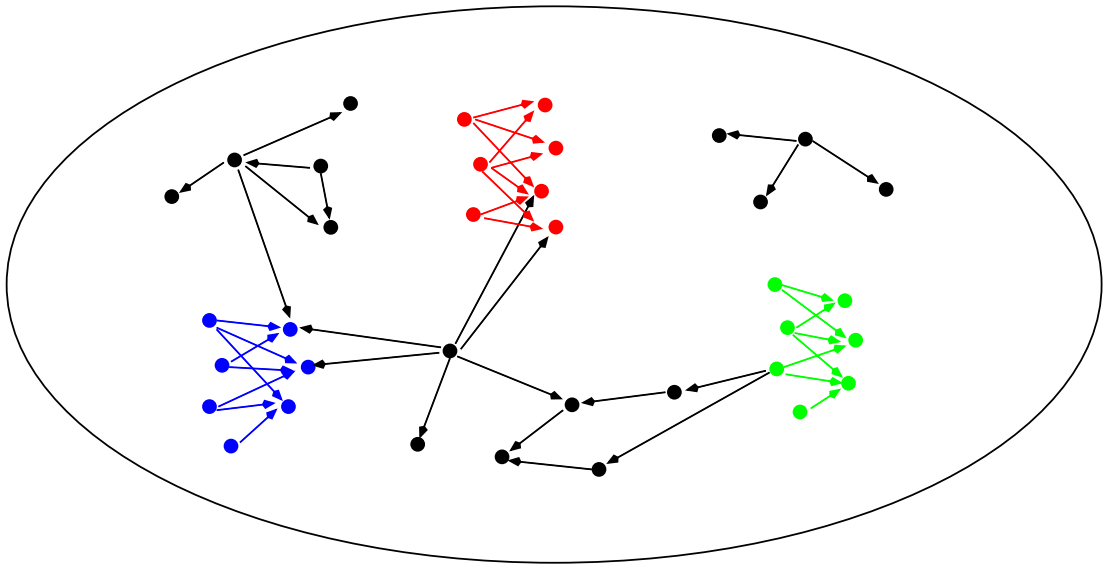
- **Formulation related to Pinski-Narin influence weights.**

. . . Google

How is authority conferred on the Web?

- **Authority-to-authority? Through a layer of hub pages?**

  (CiteSeer, Teoma, . . .)



Scan index for "signatures" of Web communities. [Kumar et al. 1999]:

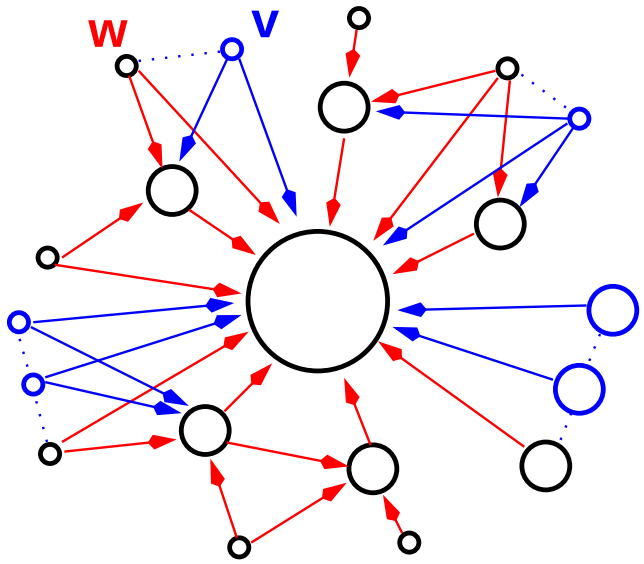
- **Expand small complete bipartite subgraphs to hubs/authorites.**
- **Can discover emerging cyber-communities (over 100,000 to date).**

  E.g.: Alumni of Delta Sigma Phi fraternity.

  Resources on Australian fire brigades.

  People concerned with oil spills off the Japanese coast.

- **How do we represent and navigate such a structure?**



Evolving network [Kumar et al.], [Barabasi-Albert], [Aiello-Chung-Lu]

- **Nodes join the network one at a time.**
- **v arrives: chooses node w at random; copies subset of out-links.**
- **The rich-get-richer dynamics of imitation.**

Theorem:

(a) Fraction of nodes with n in-links is $1/n^{\alpha}$.

(b) With high probability, many complete bipartite subgraphs.

Further Analysis

- **[Achlioptas-Azar-Fiat-Karlin-McSherry-Saia, 2000 and 2001]**

  Generative model for networks, where each node has hidden "hub importance" and "authority importance"

  Can prove that hub/authority computation approximately recovers these hidden values.

- **[Borodin-Roberts-Rosenthal-Tsaparas 2001]**

  Identification of principles underlying page ranking algorithms.

  Evaluation of algorithms according to these principles.

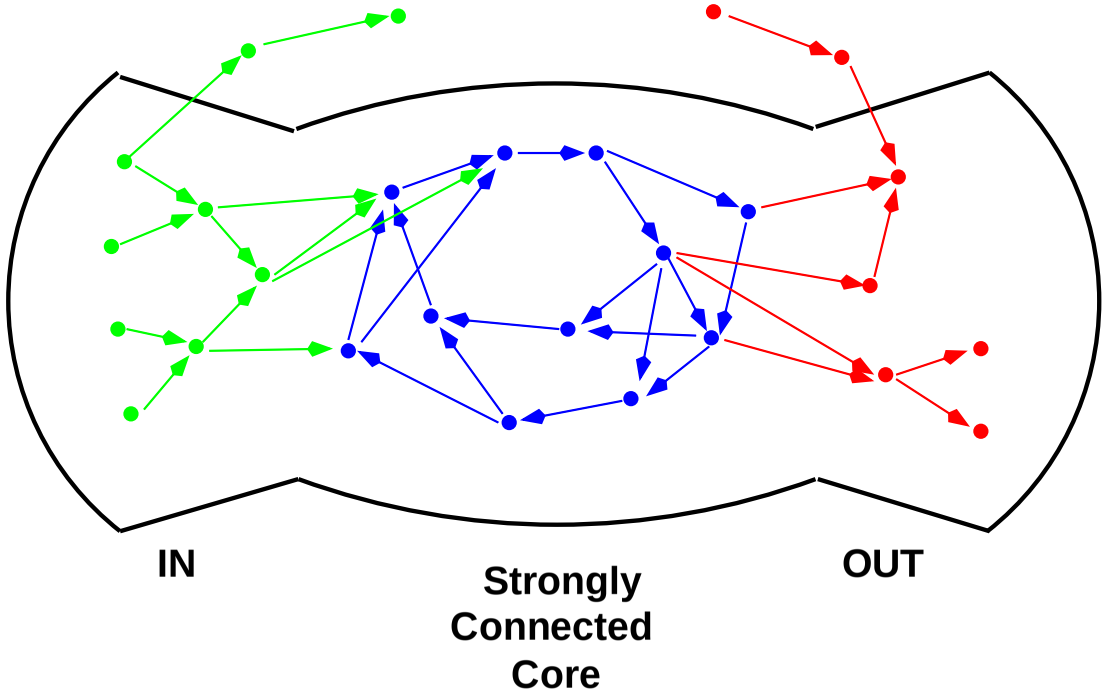
- **[Ng-Zheng-Jordan 2001]**

  Stability as a goal in the design of page ranking algorithms.

- **Consensus on a generative model for networks?**

  Other meaningful graph-theoretic structures in the Web's hyperlinks?

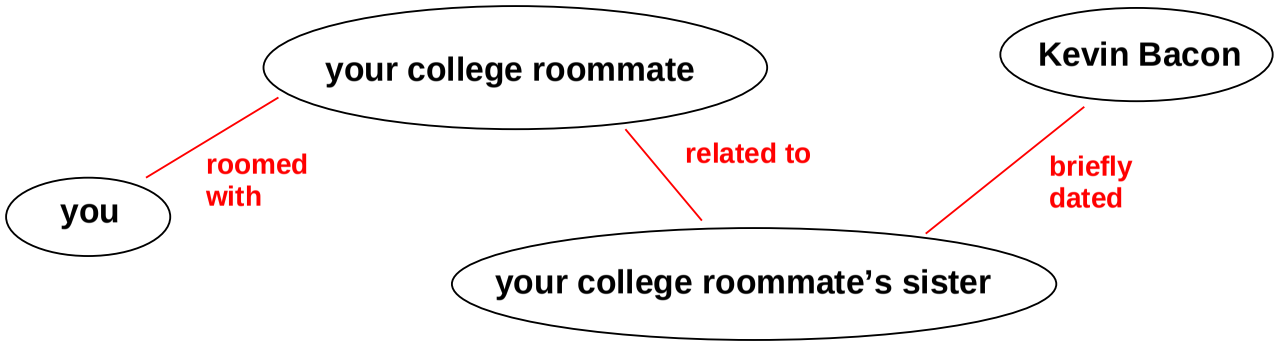
Global Structure



[Broder et al. 2000]

- **Decompose Web graph into giant strongly connected component, in-set, out-set, and a collection of "tendrils."**
- **Distances in strongly connected core very small: avg. 16-20 links. A "small-world" property [Albert-Jeong-Barabasi 1999].**

The Small-World Phenomenon



"It's a small world"

- **Why do strangers so often discover they have a friend in common? Or a short chain of friends "connecting" them?**
- **What does it say about our network of acquaintances?**

Network whose nodes are people, and links joining people who know each other on a first-name basis.

Are most pairs of people really linked by short paths?

Seems to require experimental validation. But how?

Stanley Milgram's Experiment (1967)

(1) Pick a source person in Nebraska and target in Massachusetts.

(2) Tell the source basic information about the target:

name, address, occupation.

(3) Rules for the source person:

Send the letter to someone you know on a first-name basis, with the goal of reaching the target in as few steps as possible.

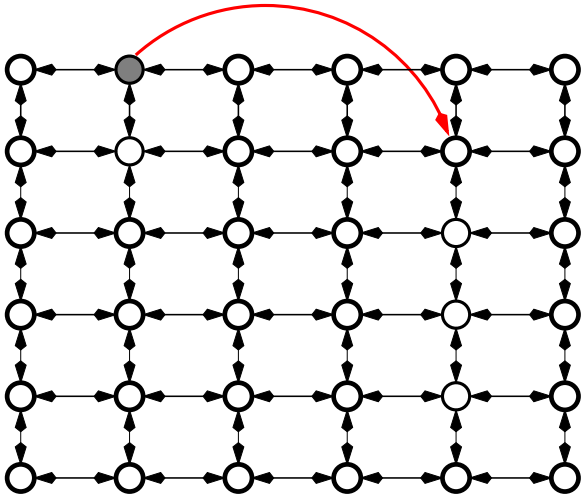
(4) All future recipients in the chain get same information and instructions, plus history.

(5) Continue until the target receives the letter.

Over chains that completed, average number of intermediate steps was between 5 and 6

$\longrightarrow$ **"six degrees of separation."**

A Small-World Network Model



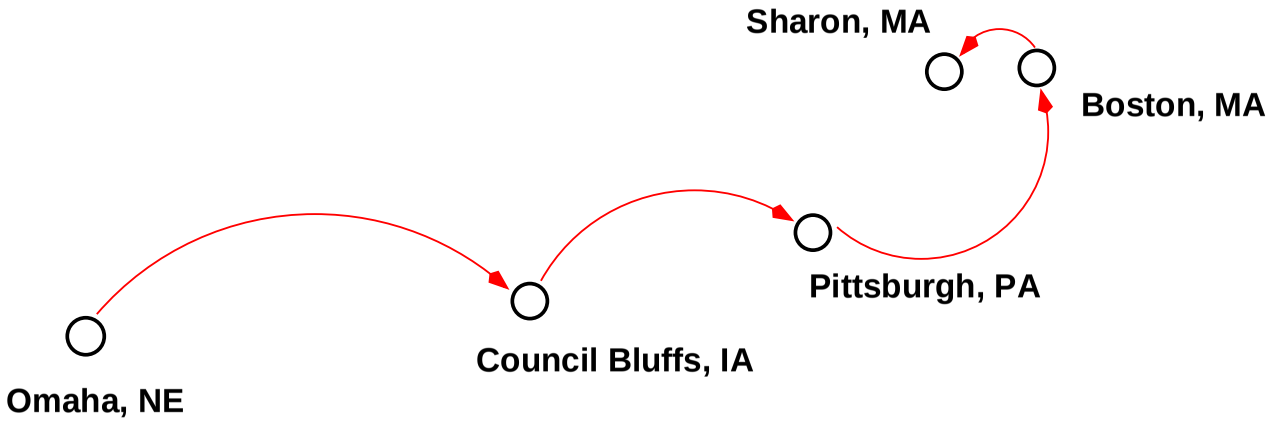
A class of networks with orderly local structure and small diameter

[Watts-Strogatz 1998].

- **Start with structured lattice network (e.g. 2-dimensional).**
- **Add a small number of random links (e.g. 1 per node).**
- **Diameter drops very quickly, while local neighborhoods remain "clustered." (See [Bollobás-Chung 1988])**

Modeling low-diameter networks as a superposition of two networks.

An Algorithmic Question



- **Why should pairs of strangers be able to find short chains of acquaintances linking them together?**
- **What does it say about our network of acquaintances?**

Navigational cues in an exponentially branching world?

The network must contain a "gradient" that guides you toward a target.

▷ **Goal: Characterize what is possible with local information.**

[Kleinberg 2000].

Decentralized Algorithms

Current message holder knows grid structure, destination, path so far.

Long-range contacts of node v only known if v has touched message.

Decentralized algorithm with delivery time polynomial in $\log n$?

(Delivery time = expected number of steps in random network.)

People were very successful at finding short paths.

Watts-Strogatz framework ideal as simple model for asking why:

- **Need a network that is partially known and partially unknown.**
- **Known part has high diameter.**
- **Full network (known + unknown) has low diameter.**
- **Structure of known part provides cues for using unknown part.**

An Impossibility Result

Theorem:

The delivery time of every decentalized algorithm in the Watts-Strogatz network is at least $\varepsilon n^{2/3}$

(for a constant ε).

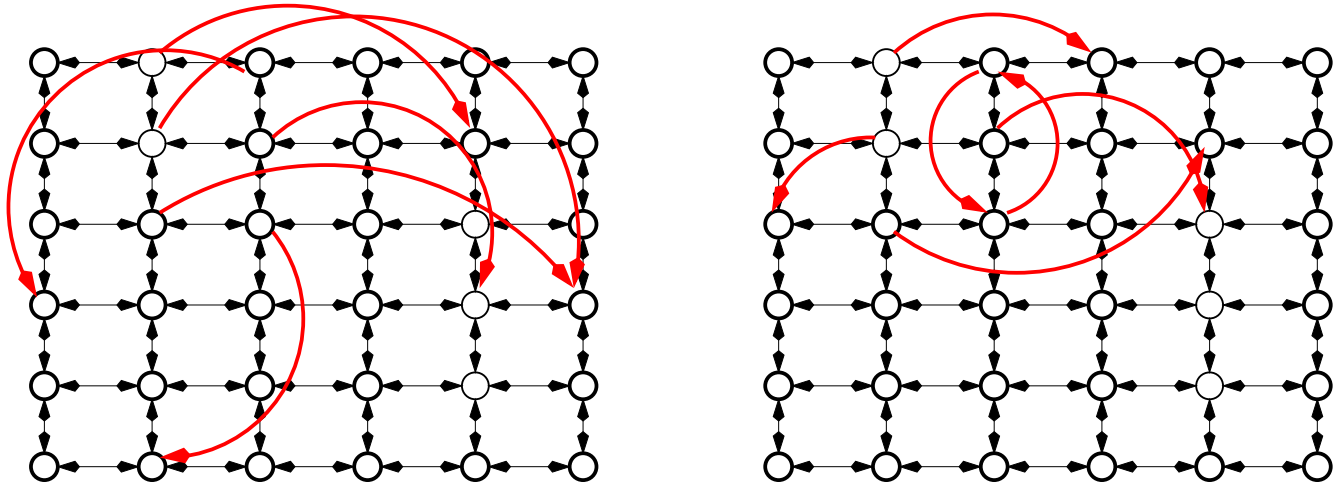
In contrast, network has diameter polynomial in $\log n$

with high probability ...

Despite the low diameter, there is no "gradient" to guide the message.

Can we find a model that has such a gradient?

Generalizing the Network Model



$n \times n$ **grid and nearest-neighbor links as before.**

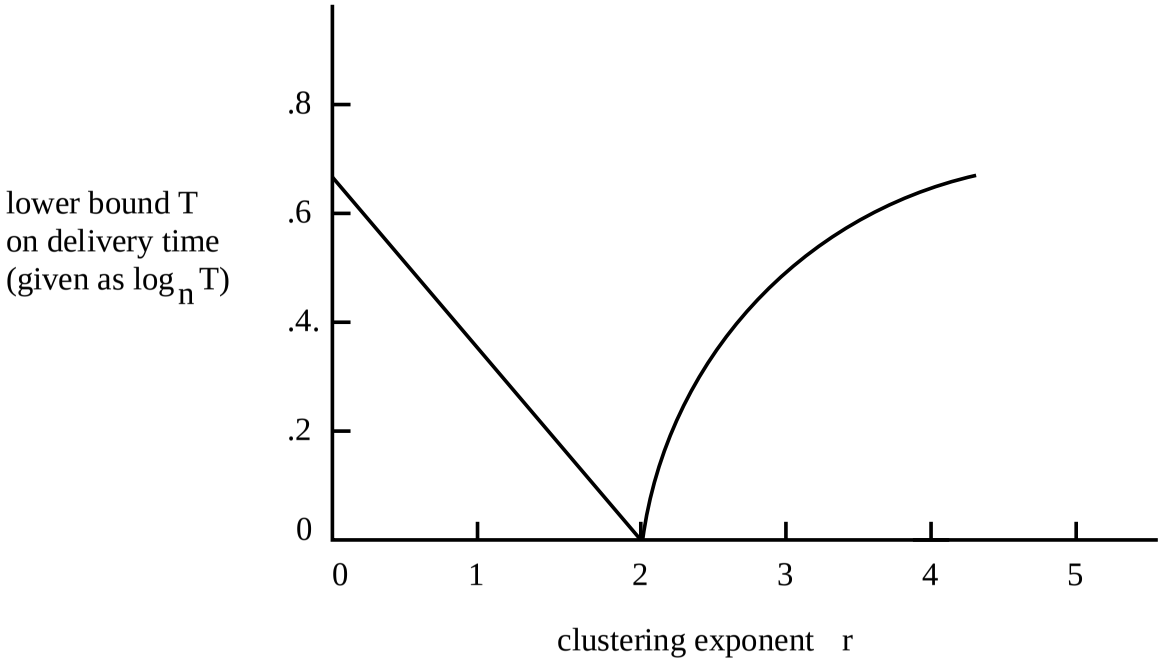
Clustering exponent r**.**

- **For each node** v**, add directed link to random long-range contact.**
- **Choose** w **as the contact with probability proportional to** $d(v,w)^{-r}$ **where** $d(v,w)$ **is the lattice distance from** v **to** w**.**

The Inverse r^{th}**-power model:**

People are more likely to have a nearby contact.

A Full Characterization



(a) $r = 2$**: There is a decentralized algorithm with delivery time proportional to** $\log^2 n$.

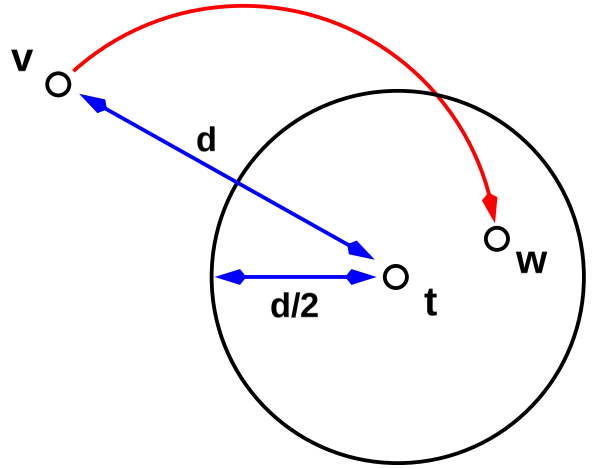
(b) $r < 2$**: Any decentralized alg. has delivery time** $\geq \varepsilon_r n^{(2-r)/3}$.

(c) $r > 2$**: Any decentralized alg. has delivery time** $\geq \varepsilon_r n^{(r-2)/(r-1)}$.

The Inverse-Square Network

When $r = 2$, the greedy algorithm is effective:

always forward to contact closest to target.



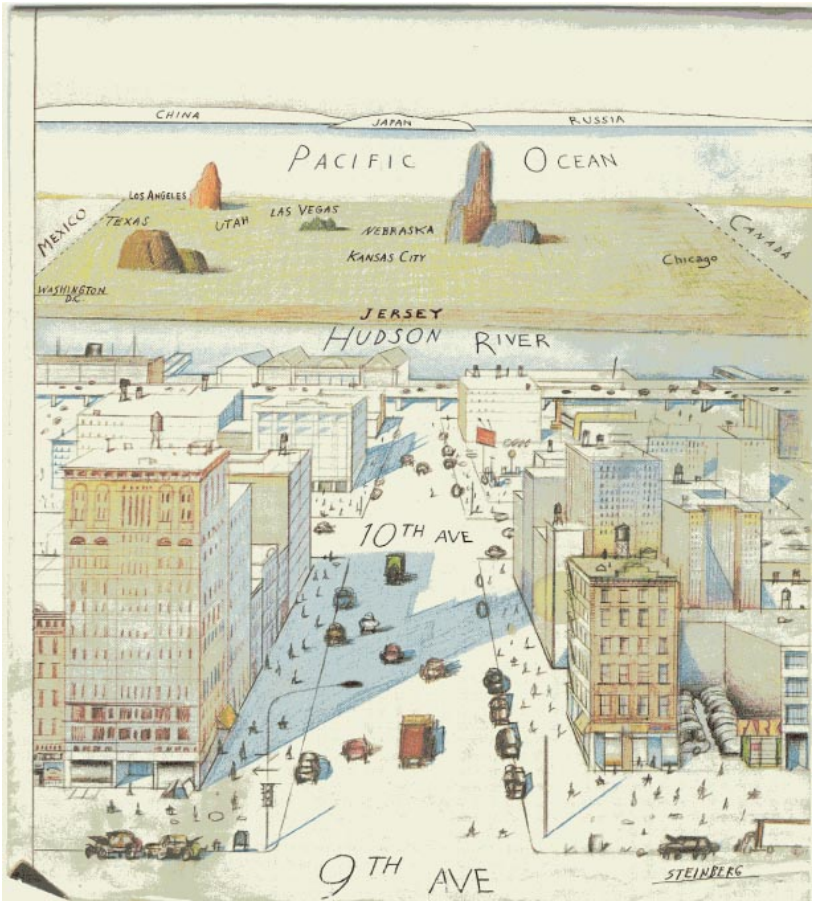
Key idea in the analysis:

In any step, there is a probability $\geq \varepsilon / \log n$

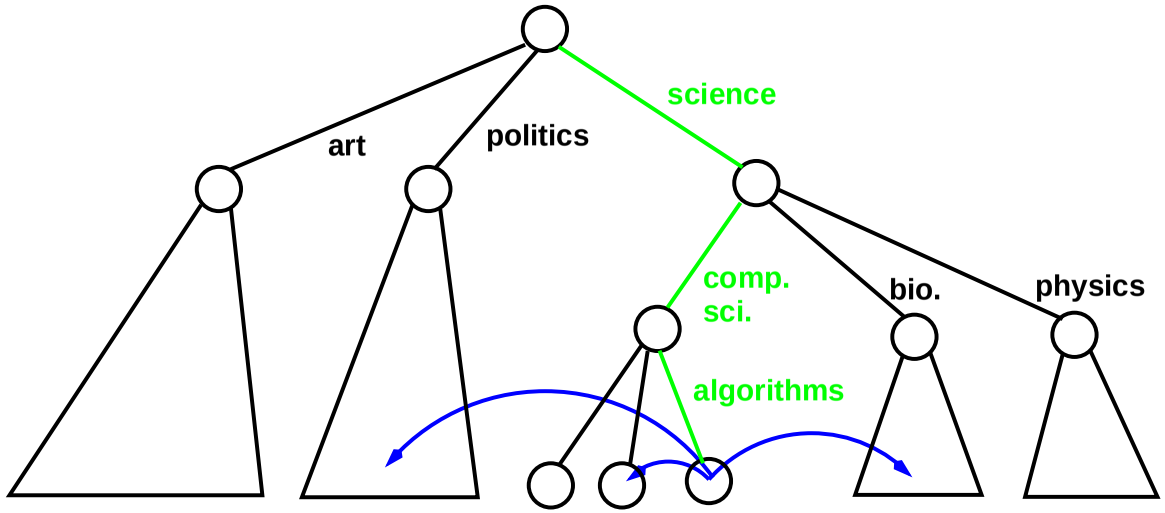
that the message's distance to the target will be halved.

Why an Inverse-Square Law?

- **Exponentially layered "distance scales" around nodes.**
  $[1, 2],\ [2, 4], \ldots\ [2^j, 2^{j+1}], \ldots$
- **When $r = 2$, nodes have roughly same proportion of links to each distance scale.**
  (e.g. [Berners-Lee 1999])
- **$r < 2$: too long-range.**
  $r > 2$: too short-range.



An Analogue for Links on the Web



Instead of a lattice, suppose underlying model is a topic hierarchy.

- **Pages reside at leaf nodes. Links created by random process:**
  Each page chooses ancestor at random (i.e. generalization)
  then chooses a leaf at random in that ancestor's subtree.
- **Analogue of small-world result:**
  Best navigation when ancestor chosen uniformly at random.
- **Open Directory measurements, focused crawling [Menczer 2002].**

Reflections

- **Decentralized peer-to-peer networks:**
  a small-world navigation problem [Gnutella, Freenet].
- **Consensus on generative models for networks?**
- **Usage, traffic, and temporal phenomena?**
- **Visualization and partial visualization of complex networks?**
- **The structures that grow up around information:**
  Many of these networks were already there.
  We just had to start exploring.